STAGE 1 MATHEMATICS

ASSESSMENT TYPE 2: MATHEMATICAL INVESTIGATION

ASTEROIDS

Task

Asteroids is a classic video game from the late 1970s that requires players to manoeuvre their way through an asteroid storm, whilst destroying the asteroids with missiles.

To move their rocket ship, players can apply forward or reverse thrust, however the velocity from the thrust is added to any existing velocity to determine the new velocity. In effect the movement of the rocket can be determined using vector operations.

In video games, distances are measured in pixels. For this assignment we will assume that the game is played in an area measuring 800 pixels wide, by 600 pixels high. We will also assume that the origin (0,0) is at the bottom left hand corner of the screen.

Step 1: Fire when ready

When a missile is fired from a moving rocket ship, its motion is governed by the initial velocity of the missile, together with the velocity of the moving ship at the time that the missile is fired. Vector operations can be used to determine the resulting velocity.

A rocket ship is moving at 50 pixels/sec in a direction of 75°T. A missile is fired from the ship with a velocity of 40 pixels/sec in a direction of 270°T.

a) Draw two vectors $v_1$and v_2 representing the initial movement of the rocket ship and the missile.
b) Combine the vector graphically and determine the magnitude and direction of the resultant vector, v_f, using trigonometry.
c) Convert the vectors v_1, v_2 and v_f into component form.
d) Add the vectors v_1 and v_2, and then compare the resultant vector to the vector v_f calculated in (b).

Step 2: Couldn't hit the side of a barn

A missile is fired from position $A(30,40)$ with a velocity vector $[50,30]$. An asteroid, with initial position $B(400,250)$ has a velocity vector $[-20,-30]$. Complete the following steps to determine the minimal separation between the missile and asteroid.

a) Calculate the speed of the missile (in pixels/sec).
b) Calculate the speed of the asteroid (in pixels/sec).
c) Explain why the position of the missile at time t sec is given by
$$r_A = [30,40] + t[50,30]$$
d) Explain why the position of the asteroid at time t sec is given by
$$r_B = [400,250] + t[-20,-30]$$
e) Find the position vector of B relative to A, i.e. $\overrightarrow{AB}$
f) Use (e) to show that if d is the distance between the missile and asteroid at any time t, then
$$d^2 = 181000 - 77000t + 8500t^2$$
g) Show that d^2 is minimum when $t = 4.53sec$
h) Hence, find the time when d is a minimum and then find the shortest distance between the missile and asteroid.
i) If the asteroid has a diameter of 80 pixels, does the missile hit the asteroid? NOTE: the position of the asteroid is measured from its centre. You can assume that the asteroid is circular in shape.

Verify your calculations and findings using the Geogebra simulator file – asteroids_step2.ggb.

Step 3: Warning! Collision imminent

In this step you will define your own scenario in which the rocket ship collides with an asteroid.

Define initial starting positions and velocity vectors for the rocket ship and asteroid such that the asteroid will collide with the rocket ship. Assume that the asteroid is the same size as given the previous question. For simplicity assume that the rocket ship is circular with a radius of 5 pixels.

a) Define position vectors r_A and r_B for the rocket ship and asteroid.
b) Find the position vector of B relative to A, i.e. $\overrightarrow{AB}$
c) Use the position vector to define an equation for the distance between the rocket ship and the asteroid and determine the time when the distance is minimum.
d) Calculate the minimum distance and hence explain why the asteroid collides with the rocket ship.
e) At what position does the collision occur? Make sure that the collision occurs within the boundaries of the screen.

Verify your calculations and findings using a simulator in Geogebra. Your simulator should be based on the simulator used in step 2.

Step 4: More thrust Captain!

In this final step you must apply thrust (acceleration) to manoeuvre between waypoints in the asteroid field.

a) Define the starting position, *S*, of the ship
b) Define positions of two waypoints, *A* and *B*, representing points that the ship must travel to. Ensure that the three points are well spread out across the screen.
c) Generate two random numbers between 20 and 50. Divide each number by 10. The first number, t_1, will represent the time in seconds that the ship takes to travel from *S* to *A*. The second number, t_2, represents the time in seconds that the ship takes to travel from *A* to *B*.
d) Calculate the velocity vector, *v*, for the movement between *S* and *A* in t_1 seconds.

Once the ship reaches waypoint A, it must apply thrust (acceleration) to get to the second waypoint. The velocity vector *v* continues to act on the ship.

e) Calculate the acceleration vector, *a*, required to counteract the velocity vector and move the ship from *A* to *B* in t_2 seconds. Use the following motion equation:

$$s(t) = v_0 t + \frac{1}{2} a t^2$$

where
s is the displacement
v_0 is the initial velocity
a is the acceleration

Verify your calculations by modifying the Geogebra simulator file – asteroids_step4.ggb.

Conclusion

You must write a conclusion summarising your findings.

The format of an investigation report may be written or multimodal.

The investigation report should be a **maximum of 8 pages** if written, or the equivalent in multimodal form.

Your report on the mathematical investigation should include the following:

- an outline of the problem and context
- the method required to find a solution, in terms of the mathematical model or strategy used
- the application of the mathematical model or strategy, including:
  - relevant data and/or information
  - mathematical calculations and results, using appropriate representations
  - the analysis and interpretation of results, including consideration of the reasonableness and limitations of the results
- the results and conclusions in the context of the problem
- a bibliography and appendices, as appropriate.

Performance Standards for Stage 1 Mathematics

	Concepts and Techniques	Reasoning and Communication
A	Comprehensive knowledge and understanding of concepts and relationships. Highly effective selection and application of mathematical techniques and algorithms to find efficient and accurate solutions to routine and complex problems in a variety of contexts. Successful development and application of mathematical models to find concise and accurate solutions. Appropriate and effective use of electronic technology to find accurate solutions to routine and complex problems.	Comprehensive interpretation of mathematical results in the context of the problem. Drawing logical conclusions from mathematical results, with a comprehensive understanding of their reasonableness and limitations. Proficient and accurate use of appropriate mathematical notation, representations, and terminology. Highly effective communication of mathematical ideas and reasoning to develop logical and concise arguments. Effective development and testing of valid conjectures.
B	Some depth of knowledge and understanding of concepts and relationships. Mostly effective selection and application of mathematical techniques and algorithms to find mostly accurate solutions to routine and some complex problems in a variety of contexts. Some development and successful application of mathematical models to find mostly accurate solutions. Mostly appropriate and effective use of electronic technology to find mostly accurate solutions to routine and some complex problems.	Mostly appropriate interpretation of mathematical results in the context of the problem. Drawing mostly logical conclusions from mathematical results, with some depth of understanding of their reasonableness and limitations. Mostly accurate use of appropriate mathematical notation, representations, and terminology. Mostly effective communication of mathematical ideas and reasoning to develop mostly logical arguments. Mostly effective development and testing of valid conjectures.
C	Generally competent knowledge and understanding of concepts and relationships. Generally effective selection and application of mathematical techniques and algorithms to find mostly accurate solutions to routine problems in a variety of contexts. Successful application of mathematical models to find generally accurate solutions. Generally appropriate and effective use of electronic technology to find mostly accurate solutions to routine problems.	Generally appropriate interpretation of mathematical results in the context of the problem. Drawing some logical conclusions from mathematical results, with some understanding of their reasonableness and limitations. Generally appropriate use of mathematical notation, representations, and terminology, with reasonable accuracy. Generally effective communication of mathematical ideas and reasoning to develop some logical arguments. Development and testing of generally valid conjectures.
D	Basic knowledge and some understanding of concepts and relationships. Some selection and application of mathematical techniques and algorithms to find some accurate solutions to routine problems in some contexts. Some application of mathematical models to find some accurate or partially accurate solutions. Some appropriate use of electronic technology to find some accurate solutions to routine problems.	Some interpretation of mathematical results. Drawing some conclusions from mathematical results, with some awareness of their reasonableness or limitations. Some appropriate use of mathematical notation, representations, and terminology, with some accuracy. Some communication of mathematical ideas, with attempted reasoning and/or arguments. Attempted development or testing of a reasonable conjecture.
E	Limited knowledge or understanding of concepts and relationships. Attempted selection and limited application of mathematical techniques or algorithms, with limited accuracy in solving routine problems. Attempted application of mathematical models, with limited accuracy. Attempted use of electronic technology, with limited accuracy in solving routine problems.	Limited interpretation of mathematical results. Limited understanding of the meaning of mathematical results, their reasonableness or limitations. Limited use of appropriate mathematical notation, representations, or terminology, with limited accuracy. Attempted communication of mathematical ideas, with limited reasoning. Limited attempt to develop or test a conjecture.